Lattice measurement of the rescaling of the scalar condensate

P. Cea a,c, M. Consoli b, and L. Cosmai a,*

^aINFN - Sezione di Bari, I-70126 Bari, Italy ^bINFN - Sezione di Catania, I-70126 Catania, Italy ^cDipartimento di Fisica, Univ. Bari, I-70126 Bari, Italy

Abstract

We have determined the rescaling of the scalar condensate $Z \equiv Z_{\varphi}$ near the critical line of a 4D Ising model. Our lattice data, supporting previous numerical indications, confirm the behaviour $Z_{\varphi} \sim \ln(\text{cutoff})$. This result is predicted in an alternative description of symmetry breaking where there are no upper bounds on the Higgs boson mass from 'triviality'.

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There are many computational and analytical evidences pointing towards the 'triviality' of Φ^4 theories in 3+1 dimensions [1,2], though a rigorous proof is still lacking. Nevertheless these theories continue to be useful and play an important role for unified model of electroweak interactions. The conventional view, when used in the Standard Model, leads to predict a proportionality relationship between the squared Higgs boson mass m_H^2 and the known weak scale v_R (246 GeV) through the renormalized scalar self-coupling $g_R \sim 1/\ln \Lambda$. In this picture, where $m_H^2 \sim g_R v_R^2$, the ratio m_H^2/v_R^2 is a cutoff dependent quantity that becomes smaller and smaller when Λ is made larger and larger.

This usual interpretation of triviality has important phenomenological implications. For instance, a precise measurement of m_H , say $m_H = 760 \pm 21$ GeV, would constrain the cutoff Λ to be smaller than 2 TeV leading to predict the existence of 'new physics' at that energy scale.

^{*} email: Leonardo.Cosmai@ba.infn.it

On the other hand there is another possible interpretation of triviality, suggested in a series of papers [3,4,5]. In this alternative approach triviality and spontaneous symmetry breaking can coexist for arbitrarily large values of the cutoff Λ . The essential point is that the 'Higgs condensate' and its quantum fluctuations undergo different rescalings when changing the ultraviolet cutoff. Therefore, the relation between m_H and the physical v_R is not the same as in perturbation theory.

To remind the basic issue we observe that, beyond perturbation theory, in a broken-symmetry phase, there are two different definitions of the field rescaling: a rescaling of the 'condensate', say $Z \equiv Z_{\varphi}$, and a rescaling of the fluctuations, say $Z \equiv Z_{\text{prop}}$. To this end, let us consider a one-component scalar theory and introduce the bare expectation value $v_B = \langle \Phi_{\text{latt}} \rangle$ associated with the 'lattice' field as defined at the cutoff scale. By $Z \equiv Z_{\varphi}$ we mean the rescaling that is needed to obtain the physical vacuum field $v_R = v_B / \sqrt{Z_{\varphi}}$. By 'physical' we mean that the second derivative of the effective potential $V_{\text{eff}}''(\varphi_R)$, evaluated at the rescaled field $\varphi_R = \pm v_R$, is precisely given by m_H^2 . Since the second derivative of the effective potential $V_{\text{eff}}''(\varphi_B)$, evaluated at the bare field $\varphi_B = \pm v_B$, is the bare zero-four-momentum two-point function, this standard definition is equivalent to define Z_{φ} as:

$$Z_{\varphi} = m_H^2 \chi_2(0) \,, \tag{1}$$

where $\chi_2(0)$ is the bare zero-momentum susceptibility. On the other hand, $Z \equiv Z_{\text{prop}}$ is determined from the residue of the connected propagator on its mass shell. Assuming 'triviality' and the Kállen-Lehmann representation for the shifted quantum field, one predicts $Z_{\text{prop}} \to 1$ when approaching the continuum theory.

Now, in the standard approach one assumes $Z_{\varphi} = Z_{\text{prop}}$ (up to small perturbative corrections). However, in the different interpretation of triviality of Refs. [3,4,5] although $Z_{\text{prop}} \to 1$, as in leading-order perturbation theory, $Z_{\varphi} \sim \ln \Lambda$ is fully non perturbative and diverges in the continuum limit. In this case, in order to obtain v_R from the bare v_B one has to apply a non-trivial correction. As a result, m_H and v_R now scale uniformly in the continuum limit, and the ratio $C = m_H/v_R$ becomes a cutoff-independent quantity.

To check this alternative picture against the generally accepted point of view, one can run numerical simulations of the theory and compare the scaling properties of $Z = Z_{\varphi}$ with those of $Z = Z_{\text{prop}}$. If the standard interpretation is correct, the lattice data for Z_{φ} should unambiguously approach unity when taking the continuum limit.

In this respect, we observe that numerical evidence for different cutoff dependencies of Z_{φ} and Z_{prop} has already been reported [6,7,8]. In those calculations, one was fitting the lattice data for the connected propagator to the (lattice

Table 1 We compare our determinations of $\langle |\phi| \rangle$ and $\chi_{\rm latt}$ for given κ with corresponding determinations found in the literature [9]. In the algorithm column, 'S-W' stands for the Swendsen-Wang algorithm, while 'W' stands for the Wolff algorithm.

κ	lattice	algorithm	$\langle \phi \rangle$	$\chi_{ m latt}$
0.074	$20^3 \times 24$	W		142.21 (1.11)
0.074	$20^3 \times 24$	Ref.[10]		142.6 (8)
0.077	32^{4}	S-W	0.38951(1)	18.21(4)
0.077	16^{4}	Ref.[9]	0.38947(2)	18.18(2)
0.076	20^{4}	W	0.30165(8)	37.59(31)
0.076	20^{4}	Ref.[9]	0.30158(2)	37.85(6)

version of the) two-parameter form

$$G_{\rm fit}(p) = \frac{Z_{\rm prop}}{p^2 + m_{\rm latt}^2}.$$
 (2)

After computing the zero-momentum susceptibility $\chi_{\rm latt}$, it was possible to compare the value of $Z_{\varphi} \equiv m_{\rm latt}^2 \chi_{\rm latt}$ with the fitted $Z_{\rm prop}$, both in the symmetric and broken phases. While no difference was found in the symmetric phase, Z_{φ} and $Z_{\rm prop}$ were found to be sizeably different in the broken phase. In fact, $Z_{\rm prop}$ was very slowly varying and steadily approaching unity from below in the continuum limit. On the other hand, Z_{φ} was found to rapidly increase *above* unity in the same limit consistently with the logarithmic trend $Z_{\varphi} \sim \ln \Lambda$ predicted in Refs. [3,4,5].

A possible objection to this strategy is that the two-parameter form Eq. (2), although providing a good description of the lattice data, neglects higher-order corrections to the structure of the propagator. As a consequence, one might object that the extraction of the various parameters is affected in an uncontrolled way. For this reason, we have decided to change strategy by performing a new set of lattice calculations. Rather than studying the propagator, we have addressed the model-independent lattice measurement of the susceptibility. In this way, assuming the mass values from perturbation theory, one can obtain a precise determination of Z_{φ} to be compared with the perturbative predictions.

Our numerical simulations were performed in the Ising limit where a one-component $(\lambda \Phi^4)_4$ theory becomes

$$S_{\text{Ising}} = -\kappa \sum_{x} \sum_{\mu} \left[\phi(x + \hat{e}_{\mu})\phi(x) + \phi(x - \hat{e}_{\mu})\phi(x) \right]$$
 (3)

and $\phi(x)$ takes only the values ± 1 (in an infinite lattice, the broken phase is found for $\kappa > 0.07475$). Using the Swendsen-Wang and Wolff cluster algo-

Table 2 The details of the lattice simulations for each κ corresponding to $m_{\rm input}$. In the algorithm column, 'S-W' stands for the Swendsen-Wang algorithm [11], while 'W' stands for the Wolff algorithm [12]. 'Ksweeps' stands for sweeps multiplied by 10^3 .

$m_{ m input}$	κ	lattice	algorithm	Ksweeps	$\chi_{ m latt}$
0.4	0.0759	32^{4}	S-W	1750	$41.714 \ (0.132)$
0.4	0.0759	48^{4}	W	60	$41.948 \ (0.927)$
0.35	0.075628	48^{4}	W	130	58.699 (0.420)
0.3	0.0754	32^{4}	S-W	345	87.449 (0.758)
0.3	0.0754	48^{4}	W	406	87.821 (0.555)
0.275	0.075313	48^{4}	W	53	104.156 (1.305)
0.25	0.075231	60^{4}	W	42	130.798 (1.369)
0.2	0.0751	48^{4}	W	27	203.828 (3.058)
0.2	0.0751	52^{4}	W	48	201.191 (6.140)
0.2	0.0751	60^{4}	W	7	202.398 (8.614)
0.15	0.074968	68^{4}	W	25	460.199 (4.884)
0.1	0.0749	68^{4}	W	24	1125.444 (36.365)
0.1	0.0749	72^{4}	W	8	1140.880 (39.025)

rithms we have computed the bare magnetization:

$$v_B = \langle |\phi| \rangle$$
 , $\phi \equiv \sum_x \phi(x)/L^4$ (4)

(where ϕ is the average field for each lattice configuration) and the zero-momentum susceptibility:

$$\chi_{\text{latt}} = L^4 \left[\left\langle |\phi|^2 \right\rangle - \left\langle |\phi| \right\rangle^2 \right]. \tag{5}$$

We used different lattice sizes at each value of κ to have a check of the finitesize effects. Statistical errors have been estimated using the jackknife. Pseudorandom numbers have been generated using RANLUX algorithm [13,14,15] with the highest possible 'luxury'. As a check of the goodness of our simulations, we show in Table 1 the comparison with previous determinations of $\langle |\phi| \rangle$ and χ_{latt} obtained by other authors [9]).

As anticipated, rather than computing the Higgs mass on the lattice we shall use the perturbative predictions for its value and adopt the Lüscher-Weisz scheme [16]. To this end, we shall denote by $m_{\rm input}$ the value of the parameter m_R reported in the first column of Table 3 in Ref. [16] for any value of κ (the Ising limit corresponding to the value of the other parameter $\bar{\lambda} = 1$).

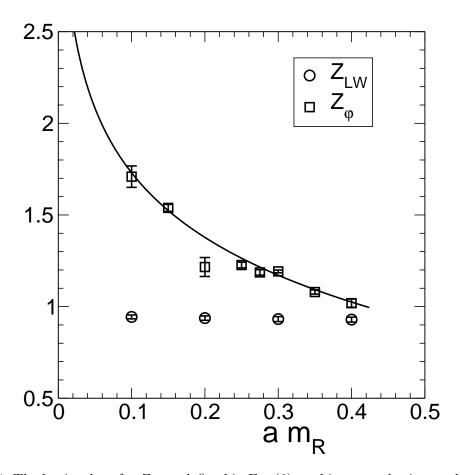


Fig. 1. The lattice data for Z_{φ} , as defined in Eq. (6), and its perturbative prediction Z_{LW} versus $m_{\text{input}} = a m_R$. The solid line is a fit to the form Eq. (8) with B = 0.50.

Our data for χ_{latt} at various κ are reported in Table 2 for the range $0.1 \leq m_{\text{input}} \leq 0.4$ (the relevant κ 's for $m_{\text{input}} = 0.15, 0.25, 0.275, 0.35$ have been determined through a numerical interpolation of the data shown in the Lüscher-Weisz Table). At this point, we can compare the quantity

$$Z_{\varphi} \equiv 2\kappa m_{\rm input}^2 \chi_{\rm latt} \tag{6}$$

with the perturbative determination

$$Z_{\rm LW} \equiv 2\kappa Z_R \tag{7}$$

where Z_R is defined in the third column of Table 3 in Ref. [16].

The values of Z_{φ} and Z_{LW} are reported in Fig. 1. We fitted the values for Z_{φ} to the form $(\Lambda = \pi/a)$

$$Z_{\varphi} = B \ln \left(\Lambda / m_R \right). \tag{8}$$

As one can check, the two Z's follow completely different trends and the discrepancy becomes larger and larger when approaching the continuum limit, precisely the same trend found in Refs.[6,7,8]. This confirms that, approaching the continuum limit, the rescaling of the 'Higgs condensate' cannot be

described in perturbation theory. Notice that the lattice data are completely consistent with the prediction $Z_{\varphi} \sim \ln \Lambda$ from Refs.[3,4,5].

On the other hand, for the symmetric phase, on the base of the theoretical predictions of Refs.[3,4,5] and on the base of the numerical results of Refs.[6,7,8], we do not expect deviations from the perturbative predictions. As an additional check, we have computed the zero-momentum susceptibility for the value $\kappa = 0.0741$ that corresponds to $m_{\rm input} = 0.2$ (see Table 3 of Ref. [17]). From our value on a 32^4 lattice $\chi_{\rm latt} = 161.94 \pm 0.67$, using again Eq. (6), we obtain $Z_{\varphi} = 0.960 \pm 0.004$. When compared with the corresponding Lüscher-Weisz prediction $Z_{\rm LW} = 0.975 \pm 0.010$, this shows that, in the symmetric phase, lattice data and perturbation theory agree to good accuracy.

Let us now return to the broken phase. If the physical v_R has to be computed from the bare v_B through $Z = Z_{\varphi} \sim \ln \Lambda$, rather than through the perturbative $Z = Z_{\text{LW}} \sim 1$, one may wonder about the m_H - v_R correlation. In this case the perturbative relation [16]

$$\left[\frac{m_H}{v_R}\right]_{\text{IW}} \equiv \sqrt{\frac{g_R}{3}}.\tag{9}$$

becomes

$$\frac{m_H}{v_R} = \sqrt{\frac{g_R}{3} \frac{Z_{\varphi}}{Z_{LW}}} \equiv C \tag{10}$$

This is obtained by replacing $Z_{\text{LW}} \to Z_{\varphi}$ in Ref. [16] but correcting for the perturbative Z_{LW} introduced in the Lüscher and Weisz approach. In this way, assuming the values of g_R reported in the second column of Table 3 of Ref. [16] and using our values of Z_{φ} one gets a remarkably constant value of C. In fact, the $Z_{\varphi} \sim \ln \Lambda$ trend observed in Fig.1, compensates the $1/\ln \Lambda$ from g_R so that $C = 3.087 \pm 0.084$ turns out to be a cutoff-independent constant [18,19].

A straightforward extension to the Standard Model of this result leads to the cutoff independent value of the Higgs boson mass $m_H = 760 \pm 21$ GeV, corresponding to $v_R = 246$ GeV and to the Ising value $C = 3.087 \pm 0.084$, so that there are no upper bounds on m_H from 'triviality'. In this sense, the whole issue of the upper limits on the Higgs mass is affected suggesting the need of more extensive studies of the critical line to compare the possible values of C in the full 2-parameter Φ_4^4 , both for the single-component and four-component theory.

In any case, a value as large as $m_H = 760 \pm 21$ GeV would also be in good agreement with a recent phenomenological analysis of radiative corrections [20] that points toward substantially larger Higgs masses than previously obtained through global fits to Standard Model observables.

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